Chiral aspects of baryon structure in the quark model.

L. Ya. Glozman

Institute for Theoretical Physics, University of Graz, Universitätsplatz 5,
A-8010 Graz, Austria; e-mail: lyg@physik.kfunigraz.ac.at

Abstract

The implications of chiral symmetry for the quark model are discussed. In particular its connection with the meson-baryon approach is outlined. It is stressed that in the closure approximation, where the intermediate baryon states are used instead of the original quark basis, it is imperative to take into account the whole infinite tower of the radially excited states in order that the crucially important effects related to the short-range part of the meson-exchange interaction be preserved. It is shown that the chiral constituent quark model is able to explain the nucleon axial vector coupling constant and automatically incorporates all the necessary loop corrections.

It has been suggested some time ago [1] that the most important interactions in light and strange baryons in the low-energy regime (below the chiral symmetry breaking scale $\Lambda_\chi \sim 1\text{GeV}$) are the effective confining interaction and the short-range part of the Goldstone boson exchange (GBE) interaction between the constituent quarks, which are quasiparticles in Bogoliubov or Landau sense and related to dynamical chiral symmetry breaking. That this should be so follows from the fact that the typical momentum of valence current quarks in the nucleon is well below the chiral symmetry breaking scale, implying that the low-energy characteristics of baryons, such as their masses, should be formed by the nonperturbative QCD dynamics, that is responsible for chiral symmetry breaking and confinement, but not by the perturbative QCD degrees of freedom, which become active at much higher momentum scale.

From the microscopical side the effective meson exchange interaction between valence quarks in baryons necessarily arises from the t-channel iterations of the QCD gluodynamics which triggers the breaking of chiral symmetry [2]. To understand the structure of the nucleon in the low-energy regime it is therefore much simpler to use an effective theory, which operates with the quasiparticles (in this case constituent quarks and chiral fields), which are necessarily implied by the original nonperturbative QCD dynamics.

The short-range part of the GBE interaction can be schematically written as [1]

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[1] Through the whole paper we use pion-exchange. The transition to the whole GBE exchange within the $SU(3)_F$ limit implies a substitution of the $SU(2)$ flavor matrices by the $SU(3)$ ones.

$$- \vec{\tau}_i \cdot \vec{\tau}_j \vec{\sigma}_i \cdot \vec{\sigma}_j, \tag{1}$$

multiplied by a radial function. In the extreme case where the scale of the chiral symmetry breaking approaches infinity, the radial form of this interaction is given by the delta-function. In the case of the finite physical pion and constituent quark sizes it should be some finite function with the range Λ_χ^{-1}. A short-range force of the same type is also supplied by the vector-meson-like exchange interaction [3], which can also be considered as a representation of the correlated two-Goldstone boson exchange, since the latter has a ρ-meson pole in the t-channel [4]. The same force is also implied by the Yukawa part of the axial vector meson exchange. There are phenomenological reasons to believe that these contributions also can be important. The recent lattice results on the origin of the $N - \Delta$ splitting [5], large N_c [6] and phenomenological [7] analyses of the negative parity $L = 1$ states support the physical picture based on the interaction (1).

In a recent paper [8] Thomas and Krein have continued a critical discussion of the chiral constituent quark model. In their first paper [9] it was argued that the pion exchange diagram of Fig. 1b, which is the basis for the interaction (1), should give results in conflict with the prediction of baryon chiral perturbation theory (BChPT). The latter suggests that in the ϵ-neighbourhood of the chiral limit the leading nonanalytic contribution (LNA) to the nucleon mass (i.e. a contribution of the order $m_c^{3/2} \sim \mu^3$, where m_c and μ are current quark and pion masses, respectively) is given by

$$\delta m_N = -\frac{3}{32\pi f_\pi^2} g_A^2 \mu^3 \simeq -15 MeV, \tag{2}$$

where μ is the pion mass in the neighborhood of the chiral limit and g_A is the nucleon axial vector coupling constant in the same regime. This contribution arises from the nucleon self-energy graph of Fig. 2a, whereas the diagram of Fig. 2c does not contribute to LNA *in the chiral limit* [11]. The argument made in ref. [9] was that the Yukawa part of the pion-exchange interaction of Fig. 1b,

$$\sim +\vec{\tau}_i \cdot \vec{\tau}_j \vec{\sigma}_i \cdot \vec{\sigma}_j \mu^2 \frac{e^{-\mu r}}{r}, \tag{3}$$

would necessarily lead to a splitting of N and Δ at the order μ^3, which is equivalent to some contribution of a diagram of Fig. 1c to the nucleon self-energy at the same order. Since the Yukawa part (3) of the meson exchange interaction is firmly related to the short-range part (1) of the pion-exchange interaction, the authors of ref. [9] concluded that the splittings that arise from this interaction are in conflict with BChPT and hence wrong.

It was, however, proven in ref. [10], that the argument above is incorrect. The reason is that the short-range interaction (1) appears at the order $m_c^0 = 1$, i.e. persists in the chiral limit, while the contribution of the Yukawa part of the meson-exchange interaction appears only in subleading orders $m_c, m_c^{3/2}, ...$ and vanishes in the chiral limit $m_c = 0$. Hence, while in the chiral limit the interaction (1) does contribute to the nucleon self-energy and to the $N - \Delta$ splitting, the Yukawa part (3) does not! Therefore there is no contradiction with

BChPT. Beyond the chiral limit (i.e. for any finite pion mass μ) the interaction (3) provides a very small splitting between N and Δ. But in this regime BChPT states that there also appears a contribution from the diagrams of Fig. 2c and Fig. 2d [11,13] and therefore there is no contradiction again.

In their second paper [8] Thomas and Krein suggested another argument, namely that the baryon-self energy should be obtained only from the projection onto the intermediate baryon states and that the N - Δ splitting, calculated after performing such a projection, was very different to that one implied by Fig. 1a and Fig. 1b and by the operator (1). Below we show that this new argument does not hold. In particular we demonstrate that the infinite tower of the radially excited states must be taken in the meson-baryon basis in order to incorporate properly the crucially important effects implied by the interaction (1). We also show that the chiral constituent quark model successfully explains the nucleon axial vector coupling constant and thus correctly incorporates all the necessary loop corrections.

Consider a generic Hamiltonian of a chiral constituent quark model, which has also been used by Thomas and Krein:

$$H = H_0 + H_\pi + W, \tag{4}$$

and assume that this Hamiltonian can be treated perturbatively with respect to the pion-quark interaction W (in reality this will not be valid). In the expression above, H_0 describes the effective confinement part of the model with eigenstates $|B_\alpha^{(0)}>$ (called bare quark states in ref. [8]), H_π is the Hamiltonian for the noninteracting pion field. The surprising statement of ref. [8] is that this Hamiltonian could not be treated directly (that would indeed correspond to the sum of diagrams of Fig. 1a and Fig. 1b) but rather an insertion of a sum over intermediate baryon-pion states should be done leading to the following self-energy:

$$\Sigma(E) = \sum_n < B_0|W^+|n > \frac{1}{E - E_n} < n|W|B_0 > . \tag{5}$$

In this expression the closure representation for the intermediate pion-baryon system has been used. It is obvious, however, that the representation (5) is correct and equivalent to the initial Hamiltonian (4) only when the full (infinite) set of the intermediate states is taken into account. When the authors of ref. [8] apply this expression to N and Δ self-energies and obtain the following relations

$$M_N = M_0' - \frac{25}{2}P_{00}' - 16P_{N\Delta}, \tag{6}$$

$$M_\Delta = M_0' - 4P_{\Delta N} - \frac{25}{2}P_{00}', \tag{7}$$

they truncate the sum over n in eq. (5) to the ground states of N and Δ only. After that they compare the expressions (6) and (7) with those of [1] (based on the interaction (1))

$$M_N = M_0 - 15P_{00}^\pi, \tag{8}$$

3

$$M_\Delta = M_0 - 3P_{00}^\pi, \tag{9}$$

and conclude from the apparent difference that (8) and (9) should be incorrect. An obvious error is that they identify M_0' and P_{00}' with M_0 and P_{00}^π. The parameter M_0, however, includes both the contribution from the constituent quark mass and the confinement. The constituent quark mass, by definition, contains the pion self-energy contribution of Fig. 1a [10]. The latter contribution is ultraviolet divergent and is absorbed into constituent quark mass after renormalization. While the coefficient P_{00}' takes into account the intermediate state N in (6) and Δ in (7), the coefficient P_{00}^π implicitly includes all allowed radially excited states with all possible radial and spin-isospin quantum numbers, which can be coupled to the nucleon or delta bare ground states via the pion absorption (emission) operator. This difference is of crucial importance.

The pion-exchange interaction of Fig. 1b contains both the ultraviolet part (1), which is independent of the pion mass, and the infrared part (3). If one is interested only in the *small* effects related with the infrared part of the meson exchange interaction, as well as in the infrared contributions from the constituent quark self energy of Fig. 1 at the order μ^3, then the truncation of the intermediate states in (5) to N and Δ ground states would be approximately justified, because all other intermediate states will not contribute at this order.[2] However, this is not true for the ultraviolet part (1) of the meson exchange interaction, which produces a much more important effect for the baryon mass. In this case the truncation of the intermediate states to the ground N and Δ states leads to astray. In a perturbative treatment this is obvious from the fact that the contribution of the interaction (1) is determined by the coefficients P_{00}^π which are proportional to the square of the bare baryon wave function at *zero* separation between two constituent quarks. If one uses the closure representation, one needs a complete infinite set of all possible radially excited states with all possible spin-isospin quantum numbers that are allowed by the pion transition in order to represent correctly the contribution above. Mathematically this statement is equivalent to the need for an infinite complete basis to represent the delta function in the closure representation and any effect of the higher terms in this expansion is not smaller than the effect of the first terms. It is easy to verify that the result obtained with the ground states only, like in (6)-(7), is dramatically different from the correct one in (8)-(9).

This is also valid for the nonperturbative treatment [14], where the finite function with the range Λ_χ^{-1} must be used instead of the original delta function in (1). For example, if one uses a harmonic oscillator basis in a variational calculation one needs all states up to $20\hbar\omega$ to achieve a convergence. This is nothing else than taking into account all highly excited intermediate baryon states.

Because the identification of P_{00}', $P_{N\Delta}^\pi$ and $P_{\Delta N}^\pi$ in (6) and (7) with $P_{00}^\pi \sim 30$ MeV in (8) and (9) is erroneous, and the expressions (6) and (7) by themselves are by no means

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[2] Note again, that this is true only in the vicinity of the chiral limit. With the real value of the pion mass, there will also appear some contributions from radially excited states.

justified, the conclusion about the nucleon self-energy of $-\frac{57}{2}P'_{00} \sim 855$ MeV is also not justified. The correct nucleon self-energy *in the chiral limit* is given by $-15P_{00}$ by the expression (8) (which is due to the interaction (1)). The constituent quark self-energy is ultraviolet divergent and thus completely absorbed into the definition of the constituent quark mass after renormalization. The different ultraviolet behaviour of graphs of Fig. 1a and Fig. 1b is lost in the expressions (6) and (7) which do not distinguish between these two graphs.

One may ask, why the typical calculations along BChPT ignore all the intermediate excited states? It is because the task of BChPT is different. It considers only corrections that are implied by the nonzero mass of current quarks (pions) and all contributions of the chiral limit (which are numerically much more important) are encoded into the fitting parameters, which are the tree level baryon N and Δ masses, coupling constants and counterterms. BChPT a-priori cannot answer the question what is the origin of the $N - \Delta$ splitting. The task of the quark model is wider. It should incorporate not only small corrections from the finite pion mass, but what is much more important it should offer an explanation of the main contributions in the chiral limit.

When one uses the quark basis and the diagrams of Fig. 1 both the chiral limit and small corrections from pion mass are met properly. There are obvious advantages even for the treatment of the small corrections from the pion mass in this case. This is because it takes into account explicitly the very different infrared behaviour of the graphs of Fig. 1a and Fig. 1b. For example, with the quark basis it is manifest that the chiral log corrections ($\sim \ln \mu^2$) to the baryon mass can appear from the quark self-energy graph of Fig 1a only and they are not supplied by the meson exchange interaction of Fig. 1b [10]. This property is lost in any usual pion-baryon description (because it does not distinguish between the two graphs in Fig. 1). It guarantees that when one uses a pion-baryon basis there should be implicit strong cancellations when one considers excited intermediate states in addition to the ground states, which are ignored in the present state of BChPT.

In addition, in the large N_c limit the $SU(6)_{FS}$ symmetry of baryons becomes exact [12] (note, that this is not the case for mesons). This symmetry is also the symmetry of the nonperturbed basis in the quark model for baryons. It is crucially important to keep manifest the large N_c behaviour and the $1/N_c$ corrections within the BChPT expansion because there must be large N_c cancellations of one-loop corrections, the property which is lost in the usual formulation of BChPT. "Not including the $1/N_c$ cancellations in a systematic way gives a misleading picture of the baryon chiral expansion" [13]. These cancellations are explicit when one uses the quark basis. The pion exchange interaction of Fig. 1b contributes to the N and Δ masses at the order N_c, but its contribution to the $N - \Delta$ splitting and to the violation of $SU(6)$ appears at the order $1/N_c$, as it must be. This implies that the combination of the $SU(6)$ quark basis and the pionic contributions at the quark level correctly preserves at the same time the implications of ChPT and large N_c expansion at the orders $N_c, 1, 1/N_c$.

The next issue to be addressed is the questioning by the authors of ref. [8] about the statement in ref. [10] that the LNA should be mostly due to the constituent quark self-energy

of Fig. 1a. This self-energy implies the following contribution

$$\delta m_N = 3 \times \left\{ -\frac{3}{32\pi f_\pi^2} (g_A^q)^2 \mu^3 \right\}, \tag{10}$$

which can be easily obtained if one considers the constituent quarks to be quasifree.[3] Here g_A^q stands for the axial vector coupling of the constituent quark. Using then the well-known $SU(6)$ relation

$$g_A = 5/3 g_A^q, \tag{11}$$

one obtains a result which is very close to (2). The success of the naive quark model relation (11) suggests that there should be a deep root in it. The $SU(6)$ symmetry of the naive quark model is the exact symmetry of QCD for baryons in the large N_c limit and in the same limit the nucleon axial charge is given by the expectation value of the operator

$$\sum_{l=1}^{N_c} q_l^\dagger \left\{ \vec{\sigma} \otimes \vec{\tau} \right\} q_l, \tag{12}$$

evaluated with exact $SU(6)$ wave functions, which gives a contribution at the order N_c [12]. This procedure, upon substitution $N_c = 3$, results in relation (11).

While the relation (11), which is model independent, should be considered as a definition of the constituent quark axial charge in the large N_c limit *with all relativistic and other possible effects automatically incorporated,* it is interesting to learn whether our intuitive view of constituent quark is compatible with this relation and the empirical value for the nucleon axial vector charge, $g_A = 1.25$.

The axial vector current of the constituent quark is given as

$$A_\mu^a = i g_A^q \bar{u}(p') \gamma_\mu \gamma_5 \tau^a u(p). \tag{13}$$

In terms of Pauli spinors it reduces to [15]

$$A^{ia} = -g_A^q \sigma^i \tau^a \left\{ 1 - \frac{2}{3} \left(1 - \frac{1}{\sqrt{1 + \tilde{\mathbf{v}}^2}} \right) \right\}. \tag{14}$$

Here the "velocity" operator is defined as $\tilde{\mathbf{v}} = (\mathbf{p}' + \mathbf{p})/2m$. This "velocity" operator should not be mixed with the standard one-particle relativistic velocity $\mathbf{v}$. In the limit $\mathbf{p}' = \mathbf{p}$ they are obviously related via $\tilde{v}^2 = \frac{v^2}{1-v^2}$. Note that the expression (14) is exact and does not assume a nonrelativistic expansion of the constituent quark spinors. It is explicitly seen

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[3]In reality the constituent quark is confined and the free Green function should be substituted by the Green function of a particle in the confining potential. That will imply some small modifications of (10).

6

from this expression that the relativistic motion factor in curly brackets *can be absorbed into a definition of the axial vector charge of the constituent quark that is effectively at rest* and thus one indeed obtains the recipe (12). Assuming that the constituent quark is relativistic, $v^2 \simeq 0.5 \div 0.7$ (as it indeed follows from the semirelativistic dynamical calculations [14] with the confining interaction as the only interaction between constituent quarks), one obtains relativistic motion correction factor $0.8 \div 0.7$. This factor, combined with expression (11) gives $g_A = 5/3 \times 0.8 \div 0.7 = 1.3 \div 1.2$. In this calculation it has been assumed that in the large N_c limit the "bare" constituent quark axial vector charge g_A^q in (14) is given by 1 [16] (there are some arguments, however, that it should not necessarily be so [18]).

All correction to the expression (11) start at the order $1/N_c$ [12]. To these belong corrections due to the pion cloud of constituent quarks [17] and due to the pion exchange interaction. As explained in ref. [10] and above the pion exchange interaction, which contributes to the violation of $SU(6)$ at the order $1/N_c$, results in the configuration mixing at this order and provides thus a small renormalization of the expression (11). It also contains the $\sim \mu^3$ contribution to the nucleon mass and thus provides other small renormalization of the expression (10). The latter contributions represent effects of the quark enviroment because they are two-body contributions and are determined by the quantum numbers of a pair of quarks, but not of a single isolated quark. Since the model does inherently contain the loop contributions of Fig. 2 and is able to reproduce (at least approximately) the axial charge of the nucleon, it necessarily reproduces LNA.

Finally a comment on the remarks by Thomas and Krein [8] and Lipkin [19] regarding the failure of the prediction of the naive constituent quark model for some specific combination of Σ and Λ magnetic moments. It is not surprising that the naive constituent quark model, which ignores all effects implied by chiral symmetry dynamical breaking fails here. Within the chiral constituent quark model these implications assume the pion-loop contribution at the quark level [20] as well as the meson exchange current (MEC) contributions (see e.g. the recent papers [15,21]). The latter represent two-body effects, i.e. effects of the quark enviroment (in the language of Thomas and Krein [8] and papers cited therein). While the results of the qualitative calculations show that the effects of the MEC for some observables, specifically for the magnetic moments should be significant and work in a proper direction [15] (because, as it is well known from nuclear physics, the MEC contributions to M1 electromagnetic transitions are large), one cannot unfortunately calculate them reliably because the problem is highly relativistic and the derivation of the meson exchange current operators at the order v^2/c^2, justified in nuclear physics, cannot be considered to be quantitative in the present application. But an important issue, which we want to stress, is that the LNA contribution and other chiral corrections to magnetic moments *are taken into account* as soon as pion loops and MEC contributions are properly included.

The conclusion is that the meson-baryon or quark models which employ only the subspace of the pion-nucleon and pion-delta states, are incomplete and ignore the most important short range effects of the pion (meson) degrees of freedom for the baryon structure. If one uses the pion(meson)-baryon basis it is imperative to include the whole tower of the excited baryon states.
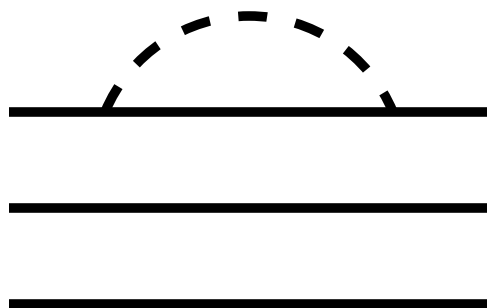
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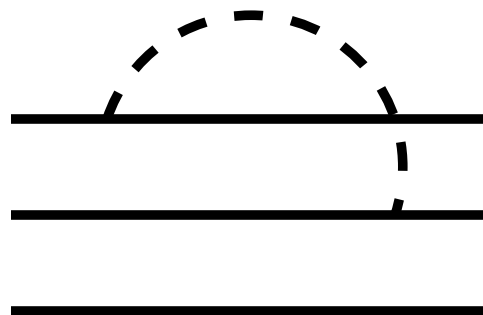
Figure captions

Fig.1 Pion loop a) and pion exchange b) contributions to the baryon mass within the chiral constituent quark model.

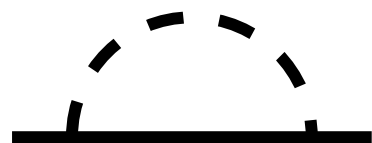Fig.2 Pion loop contributions to the baryon mass with the pion-baryon system in the intermediate state truncated to N and Δ states only.
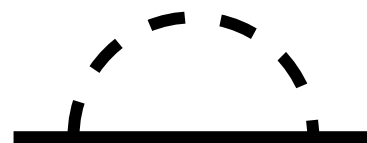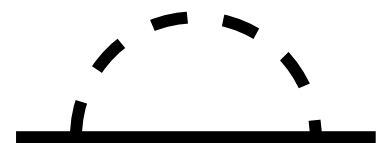
a   b

a

b

c

d